THE UNITED REPUBLIC OF TANZANIA
PRESIDENT'S OFFICE
REGIONAL ADMINISTRATION AND LOCAL GOVERNMENT

FORM SIX SPECIAL SCHOOLS SYNDICATE JOINT EXAMINATION

142/2 **ADVANCED MATHEMATICS 2**

Time: 3:00 Hrs **Monday 13-March-2023 PM**

INSTRUCTIONS:

i) This paper consist of two sections A and B
ii) Answer all questions from section A and any two questions from section B.
iii) A non – programmable calculators and mathematical tables may be used.
iv) Write your name / examination number on every page of your answer sheet(s)

SECTION A (60 Marks)
Answer All Questions in this Section

1. (a) Prove that; ${}^{n}C_{r} = {}^{n-1}C_{r} + {}^{n-1}C_{r-1}$

(b) X is a random variable with probability distribution below

X	1	2	3	4
$P(X = x)$	P	0.2	q	0.2

If the mean of this random variable is 2.6 find

(i) The valve of P and q

(ii) $Var\ (3x + 7)$

(iii) Standard deviation of x

(c) In KIZUKA village there are 800 families with 5 children each. Assuming that the probability of male birth in the village is $\frac{1}{2}$, how many families would be expected to have (i) Either 2 or 3 Boys

(ii) 5 Girls

2. (a) Let P and q be the propositions

P: Mathematics is easy

Q: Five is greater than four

Write in words the propositions

(i) $p \wedge q$

(ii) $\tilde{}\,(p \vee q)$

(iii) $(p \wedge \tilde{}q) \vee (\tilde{}p \wedge q)$

(b) Use laws of algebra of propositions to simplify as much as possible the compound statement $[p \rightarrow (q \vee \tilde{\tilde{r}})] \rightarrow (p \wedge q)$

(c) Test the validity of the following argument using the truth table

"The seedling will survive only if it rains. If seedlings survive well, animals will not die. But animals are dying. Therefore it is not raining.

3. (a) The scalar product of the vector $i + j + k$ with a vector along the sum of vectors $2i + 4j - 5k$ and $xi + 2j + 3k$ is equal to one. Find the value of x.

   (b) If O is the origin where

   $\overrightarrow{OA} = 3i + j - 2k$, $\overrightarrow{OB} = i - 2j + 3k$ and $\overrightarrow{OC} = -i + 4j + 2k$

   Calculate the area of ABC

   (c) If $\vec{a} = i + mj + 3k$ and $\vec{b} = 4i + 3j$, the projection of a and $\vec{b}$ is $^2/_3$. Find the value of m.

   (d) Using the concept of scalar product to prove cosine rule and deduce the Pythagoras formula.

4. (a) Let $Z = x + yi$ be a non zero solution of the quadratic equation $az^2 + bz + c = 0$, where a, b, c are real coefficients with $a \neq 0$ and $ac > b^2$. Express x and y in terms of the coefficients a, b and c.

   (b) Express $= \frac{10-2\sqrt{3}\,i}{1-3\sqrt{3}\,i}$, in the form $r\,(cos\theta + isin\,\theta)$. Hence find $Z^{1/4}$.

   (c) Prove that $Sin7Q = \frac{1}{64}(35Sin\theta - 21Sin\theta + 7Sin5\theta - Sin\,7\theta)$. Hence find $\int(35sin\theta - 64\sin^7\theta)d\theta$

SECTION B. (40 Marks)

Answer any two questions from this section.

5. (a) If $sin\theta = \frac{1}{2}(n + \frac{1}{n})$ Prove that $sin3\theta + \frac{1}{2}(n^3 + \frac{1}{n}) = 0$

   (b) Find x if $\tan^{-1}(2x + 1) - \tan^{-1}(2x - 1) = \tan^{-1}(\frac{1}{8})$.

   (c) Find the general solution of the equation $Sinx + Sin5x = Sin2x + Sin4x$

   (d) Find the maximum and minimum value of the expression $3sin\theta + 5Cos\theta + 7$.

   (e) (i) Approximate $\frac{1-cos2\theta}{\theta sin\theta}$ as θ is very small angle

   (ii) Given that $y = \frac{sinx-2sin2x+sin3x}{sinx+2sin2x+sin3x}$, prove that $y = -\tan^2\frac{x}{2}$

6. (a) Solve for x given $x + log(1 + 2^x) = xlog5 + log\,6$.

(b) Use formula for $\sum_{r=1}^{n} r^3$ to find $\sum_{r=7}^{20} r^3$

(c) By means of mathematical induction, prove that $3^n + 3^{n+1}$ is divisible by 12 for all positive integers.

(d) Show that $f(x)$ is divisible by $(x - a)\,(x - b)$ where $a \neq b$ then the remainder is given by $r = \left(\frac{f(a)-f(b)}{a-b}\right)x + \frac{af(b)-bf(a)}{a-b}$.

7. (a) Solve the following

(i) $\frac{1}{x}\frac{dy}{dx} = ye^{x^2} + \sqrt{y}\, e^{x^2}$

(ii) $secx\,\frac{dy}{dx} = e^{y+sinx}$

(b) Find the differential equation of the family of circles of radius 5cm and their centres lying on the x – axis.

(c) For what values of $m\ and\ n$ does $y = e^{4x} + e^{-x}$ satisfy the differential equation $\frac{d^2y}{dx^2} - m\frac{dy}{dx} + ny = 0$?

(d) Solve the differential equation $\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 2y = cosx$

(e) The rate of change of the number of coyotes $N(t)$ in a population is directly proportional to $650 - N(t)$, where t is the time in years, when t = 0 the population is 300 and when t = 2, the population has increased to 500. Find the population when t = 3.

8. (a) Prove that the equation of asymptotes to the conic section $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ are given by $y = \pm\frac{b}{a}x$

(b) Find the centre, eccentricity, foci and directrix of

$4x^2 + 9y^2 - 16x + 72y + 124$

(c) Prove that $y = -3x + b$ is a tangent to the rectangular hyperbola whose parametric coordinates are of the form $(\sqrt{3}\, t, \frac{1}{t}\sqrt{3})$

(d) The normal at point P(ap,2ap) meets $x - axis$ at G. Find the coordinates of point G. If H is the point provided PG=GH. Find the coordinate of H in terms of P and show that H lies on the parabola $y = 4a(x - 4a)$.

(e) Sketch the graph of $r = 1 + 2sin\theta$